



UNIVERSITI SAINS MALAYSIA

1st. Semester Examination
2000/2001 Academic Session

SEPTEMBER / OCTOBER 2000

EAS351/3 – Finite Element Method For Engineers

Time : [3 hours]

Instruction to candidates:-

1. This paper consists of **SEVEN** (7) questions. Answer **FIVE** (5) questions only.
2. Answers **MUST BE** written in Bahasa Malaysia.

1. (a) Clearly define the difference between a finite element method and a finite difference method. Give an example of any structures of their applications in civil engineering.

(5 marks)

- (b) A deep simply supported beam supporting a uniformly distributed load, $q = 1 \text{ N/cm}$ and it was analysed as a plane stress case. Three meshes were employed using 16, 32 and 64 elements respectively, as shown in Figure 1a . Discuss and compare the results obtained to an exact solution based on the plane elasticity theory in Table 1.

(10 marks)

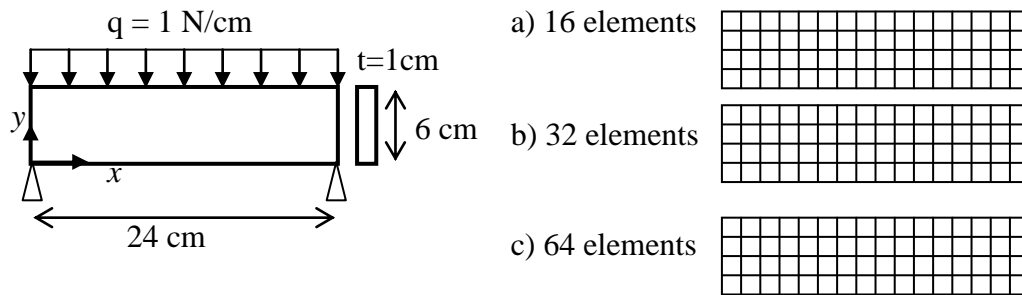


Figure 1(a)

Element	Vertical Deflection		Longitudinal stress
	Point A ($\text{cm} \times 10^{-6}$)	Point B ($\text{cm} \times 10^{-6}$)	Point C (N/cm^2)
16	782	560	10.8
32	844	605	11.9
64	861	616	12.0
Exact solution	898	645	12.2

Table 1

- (c) Element stiffness matrix \mathbf{k}_E for a linearly elastic spring with spring constant k is given as follows :

$$\mathbf{k}_E = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$$

Explain the physical meaning of the coefficients of \mathbf{k}_E .

(3 marks)

State another property of element stiffness matrix (and also unreduced structure stiffness matrix).

(2 marks)

2. (a) Clearly define the difference between a triangular and rectangular finite element for plane elasticity.

(5 marks)

- (b) A deep cantilever carrying a load of 10 kN as shown in Figure 2a. The cantilever is assumed fully fixed at the left-hand edge and is idealised as shown in Figure 2b. Given $E = 200 \text{ kN/m}^2$, $\nu = 0.3$ and the thickness, $t = 5 \text{ mm}$. Develop a stiffness matrix, $[K]^e$, for element number 1, made up by rectangular elements :

- (i) having a plane stress
(ii) having a plane strain

(15 marks)

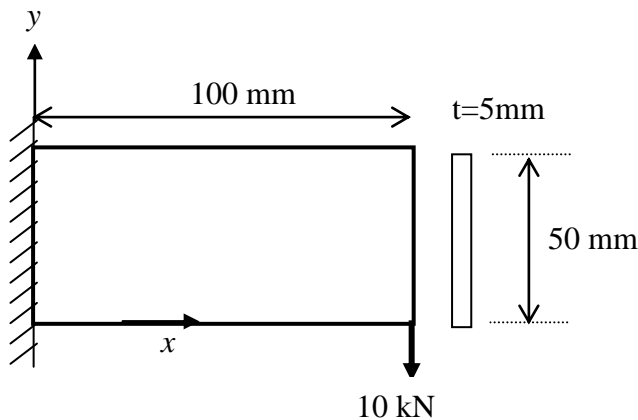


Figure 2(a)

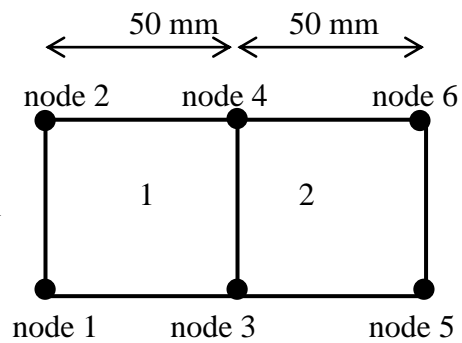


Figure 2(b)

3. (a) Show clearly a step by step to develop a stiffness matrix, $[K]^e$, for element number 1, made up by triangular elements and having a plane stress as shown in Figure 3. Given $E = 30 \times 10^6 \text{ N/m}^2$, $\nu = 0.3$ and $t = 0.25 \text{ m}$.

(15 marks)

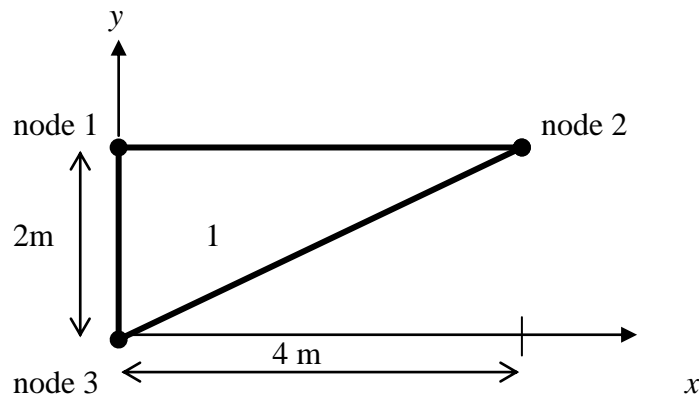


Figure 3

3. (b) If the stiffness matrix, $[K]^e$ for the triangular element in Figure 3 is given below, find the displacements, u_2, v_2 at node 2 when both node 3 and node 1 are pinned and the horizontal force at node 2, $F_{x2} = 5 \text{ kN}$.

$$[K]^e = \begin{bmatrix} 2.06 & 0 & - & 1.24 & 0 & -1.24 \\ & 0.7 & 1.44 & - & - & 0 \\ & 2 & & 0.72 & 1.44 & \\ -2.06 & 1.4 & 4.94 & - & - & 1.24 \\ & 4 & & 2.68 & 2.88 & \\ 1.24 & - & - & 8.97 & 1.44 & -8.25 \\ & 0.7 & 2.68 & & & \\ & 2 & & & & \\ 0 & - & - & 1.44 & 2.88 & 0 \\ & 1.4 & 2.88 & & & \\ & 4 & & & & \\ -1.24 & 0 & 1.24 & - & 0 & 8.25 \\ & & & 8.25 & & \end{bmatrix} \times 10^6 \text{ N/m}$$

(5 marks)

4. (a) Principle of virtual displacement (PVD) is the basis for the formulation of finite element equations. State what PVD is.

(2 marks)

- (b) Show by using principle of virtual displacement that the element stiffness equation for a linearly elastic spring element as depicted in Fig. 4 is as follows:

$$k \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u^i \\ u^j \end{Bmatrix} = \begin{Bmatrix} f^i \\ f^j \end{Bmatrix}$$

where:

k : spring constant

f^i, f^j : nodal forces at node i and j of the spring element, respectively

u^i, u^j : nodal displacement of node i and j of the spring element, respectively

(5 marks)

Figure 4

What does element stiffness equation represent ?

(1 mark)

4. (c) Figure 5 shows a system of three rigid carts interconnected by four linearly elastic springs. Load R_1 , R_2 and R_3 act on cart \leftarrow , \uparrow , and \rightarrow , respectively, where $R_1=50$, $R_2=0$ and $R_3=10$.

Figure 5

Determine :

- (i) the displacements U_1 , U_2 and U_3 of cart \leftarrow , \uparrow , and \rightarrow , respectively ; and
- (ii) the reaction at support point P

by using the finite element method.

(12 marks)

5. (a) Figure 6 shows a bar/truss element in xy-plane. The axis of bar element makes an angle θ in anti-clockwise direction with global axis x. Nodal force vectors \mathbf{f}^i and \mathbf{f}^j act at node i and j of the element, respectively. The corresponding nodal displacement vectors are \mathbf{u}^i and \mathbf{u}^j , respectively.

Figure 6

By applying principle of virtual displacement to the bar/truss element, it can be shown that the element stiffness equation in local coordinate system O'-x'y' could be written as follows :

$$\begin{Bmatrix} f^i \\ f^j \end{Bmatrix} = \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u^i \\ u^j \end{Bmatrix}$$

where :

f^i, f^j : x'-components of vectors \mathbf{f}^i and \mathbf{f}^j in local coordinate system O'-x'y', respectively

u^i, u^j : x'-components of vectors \mathbf{u}^i and \mathbf{u}^j in local coordinate system O'-x'y', respectively

E, A, L : Young's modulus, cross-sectional area and length of the element, respectively

Show that in the global coordinate system O-xy, the corresponding element stiffness equation could be expressed as follows:

(6 marks)

$$\begin{Bmatrix} F_x^i \\ F_y^i \\ F_x^j \\ F_y^j \end{Bmatrix} = \frac{EA}{L} \begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix} \begin{Bmatrix} U_x^i \\ U_y^i \\ U_x^j \\ U_y^j \end{Bmatrix}$$

where:

F_x^i, F_y^i : components of \mathbf{f}^i in global coordinate system O-xy

F_x^j, F_y^j : components of \mathbf{f}^j in global coordinate system O-xy

U_x^i, U_y^i : components of \mathbf{u}^i in global coordinate system O-xy

U_x^j, U_y^j : components of \mathbf{u}^j in global coordinate system O-xy

$c=\cos\theta$, $s=\sin\theta$

Apply equation (1) to the problem shown in Fig. 1 and show that

$$U_2 = -\frac{P}{2\sqrt{2}EA}$$

(2 marks)

Figure 7

5. (b) In finite element (displacement approach) analysis, stresses in elements are obtained by using the stress-strain relations after all the nodal displacements have been determined.

Write down the stress-strain relation for a bar/truss element (indicate clearly the meanings of symbols used in the relation).

(1 mark)

Derive the equation relating element stress in a bar/truss element with the components of nodal displacements $U_x^i, U_y^i, U_x^j, U_y^j$ in global coordinate system O-xy as shown in Fig. 8 (use the symbols L for element length and E for Young's modulus).

(5 marks)

Figure 8

- (c) A plane truss problem as shown in Figure 9 has been analysed using a finite element program. Coordinates of node \leftarrow to \downarrow and also the connectivity of the five elements of the plane truss are shown in Table 2. Nodal displacements which are obtained from the analysis are listed in Table 3.

$E = 20 \times 10^4$ for all elements
 $A = 1$ for all elements

Figure 9

Table 2

Finite element program.

Data nodal coordinates and element connectivity

NODE	X-COORDINATE	Y-COORDINATE	Z-COORDINATE
Node 1	0.00000E+00	0.00000E+00	0.00000E+00
Node 2	5.00000E-01	1.00000E+00	0.00000E+00
Node 3	1.50000E+00	1.00000E+00	0.00000E+00
Node 4	1.00000E+00	0.00000E+00	0.00000E+00

ELEMENT	NODE 1	NODE 2
Element 1	1	2
Element 2	2	3
Element 3	1	4
Element 4	2	4
Element 5	4	3

Table 3

Finite element program.

Data for nodal displacements:

NODE	X-DISPLACEMENT	Y-DISPLACEMENT	Z-DISPLACEMENT
Node 1	0.00000E+00	0.00000E+00	0.00000E+00
Node 2	6.36271E-03	3.12500E-04	0.00000E+00
Node 3	8.86271E-03	-1.20441E-02	0.00000E+00
Node 4	-1.25000E-03	0.00000E+00	0.00000E+00

By using the equation relating element stress and nodal displacements derived in 5(b) above, calculate the stresses in all five elements of the plane truss. Summarize your answers in a table.

(6 marks)

6. (a) Derive the finite difference approximation of a time derivative $\frac{\partial z}{\partial t}$ using Taylor

Series expansion for:

- (i) forward difference approximation
- (ii) central difference approximation
- (iii) backward difference approximation

(10 marks)

- (b) Derive the finite difference approximation using central difference approximation for space derivative and backward difference approximation for time derivative for the following equations:

- (i) parabolic equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

- (ii) hyperbolic equation

$$\frac{\partial u}{\partial t} + C^2 \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

(15 marks)

7. Derive the finite difference approximation of the Laplace equation and give the formulas (of the Laplace equation finite difference approximation) for Jacobi Iteration, Gauss-Seidel iteration and Successive over relaxation. Describe also the method of implementation and how the three methods of iteration differ.

Given the Laplace equation:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0$$

(25 marks)

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